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U. S. NAVAL ORDNANCE LABORATORY
White Oak, Silver Spring 19, Maryland

NAVAL ORDNANCE LABORATORY MEMORANDUM 10594
SUPPLEMENT I

4 April 1950

From: J. C. Crown
To: NOL Files
Via: Chief, Hyperballistics Division
Aeroballistic Research Department

Subj: Supersonic Nozzle Design. (Project NOL 159)

Abstract: An analytic method for designing nozzles is described herein. This method (Nilson's) allows for the nonuniformity of flow across the throat.

Foreword: This supplement covers material that was the subject of a course on nozzle design given by the author and Dr. W. H. Heybey. The data and conclusions presented here are for the use of the personnel of the Naval Ordnance Laboratory. They do not necessarily represent the final opinion of the Laboratory.

Refs: (a) Nilson, E. N. Design of an Inlet for a Two-Dimensional Supersonic Nozzle. U.A.C. Meteor Project Report No. 13 (1947). Confidential.
(b) Nilson, E. N. Analytic Correction of a Two-Dimensional Nozzle for Uniform Exhaust Flow. U.A.C. Meteor Project Report No. 15 (1948). Restricted.
(c) Scarborough, J. B. Numerical Mathematical Analysis, Baltimore, 273 (1930).

Encl: (A) Figure 1
(B) Table I

I. Nilson's Method

1. Various people have developed exact analytic methods for designing nozzles; however, the author is only sufficiently familiar with that of Nilson (references a and b) to discuss it herein.

2. Essentially his method consists of finding an exact solution for flow in the region of the throat and then converting the divergent supersonic flow thus produced into a uniform stream at the desired Mach number. This conversion into a uniform stream is similar to the conversion of source flow into a uniform stream in the Foelsch method; the difference

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between the two methods being that Foelsch assumes that he will have source flow while Nilson has an exact solution and knows what velocity distribution his contour will produce. Both consider only symmetrical nozzles.

3. More specifically, Nilson assumes a given velocity distribution along the nozzle centerline. He expresses his ordinates and velocity as infinite series, the coefficients of which he determines from the dynamical equations. This method of designing nozzles is naturally limited to cases in which the series converge within the desired accuracy; (i.e., at more or less low Mach numbers). To insure the proper convergence the present author evaluated the coefficients of the third terms; Nilson gave only the first two.

4. Let us adopt the following notation:

x, y = rectangular coordinates, X axis coincident with nozzle centerline

r, θ = magnitude and direction (relative to X axis) of velocity vector.

ξ, η = coordinate system employing equipotential line and streamlines (see reference a).

$$M = \frac{V}{c}, \qquad M^* = \frac{V}{c^*}$$

where c^* is the critical sonic speed

$$h = \frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + \frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\right]^{\frac{1}{2}\frac{\gamma+1}{\gamma-1}}$$

A bar over a variable indicates that the particular variable is taken on the nozzle centerline.

5. The solution is assumed in terms of the following infinite series:

$$x = \xi + x_2\eta^2 + x_4\eta^4 + \cdots \tag{1}$$

$$y = y_1\eta + y_3\eta^3 + y_5\eta^5 + \cdots \tag{2}$$

$$\frac{M^*}{\bar{M}^*} = 1 + m_2\eta^2 + m_4\eta^4 + \cdots \tag{3}$$

$$\theta = \theta_1\eta + \theta_3\eta^3 + \cdots \tag{4}$$

The coefficients were found to be

$$x_2 = -\frac{1}{2}\bar{h}\bar{h}' \qquad \text{(prime denotes differentiation with respect to } \xi \text{)} \tag{5}$$

$$x_4 = -\frac{\bar{h}}{6}\left[\frac{1}{2}\bar{h}\bar{h}'\bar{h}'''(\bar{M}^2-1) - \frac{\bar{h}'^3}{6} + \theta_3\right] \tag{6}$$

$$y_1 = \bar{h} \tag{7}$$

$$y_3 = \frac{\bar{h}}{6}\left[(\bar{M}^2-1)\bar{h}\bar{h}'' - \bar{h}'^2\right] \tag{8}$$

$$y_5 = \frac{\bar{h}}{5}\Big[m_4(\bar{M}^2-1) - \bar{h}'\theta_3 - \frac{1}{4}\bar{h}\bar{h}'^2\bar{h}''(\bar{M}^2-1) + \frac{1}{4}\bar{h}^2\bar{h}''^2\bar{M}^2\left(1+\frac{\gamma-1}{2}\bar{M}^2\right) + \frac{\bar{h}'^4}{24}\Big] \tag{9}$$

$$m_2 = \frac{1}{2}\bar{h}\bar{h}'' \tag{10}$$

$$m_4 = \frac{\bar{h}^2\bar{h}''^2}{4}\left(\frac{\bar{M}^2+1}{2}\right) + \frac{\bar{h}}{4}\theta_3' \tag{11}$$

$$\theta_1 = \bar{h}' \tag{12}$$

$$\theta_3 = \frac{\bar{h}}{6}\left\{\bar{h}'\bar{h}''\left[\frac{(\gamma+1)\bar{M}^4}{\bar{M}^2-1} - 1\right] + \bar{h}\bar{h}'''(\bar{M}^2-1)\right\} \tag{13}$$

6. The function $\bar{h} = \bar{h}(\xi)$ occurring in the preceding formulae is the assumed centerline velocity distribution. For a distribution $\bar{h} = 1+\xi^2$, an approximately hyperbolic contour is generated. If this flow field is now cut off along any Mach line and the flow converted into a uniform and parallel stream, there will be a discontinuity in streamline curvature at the junction of these two flows. Such a discontinuity is not desirable; but, however, what distribution must be chosen to yield streamlines possessing inflexion points is not known.

For low Mach numbers, at least, the change in curvature is not too serious providing proper discretion is used. How much curvature discontinuity can be safely used remains to be determined from experiment.

7. Using $\bar{h} = 1 + \xi^2$, we can simplify some of the coefficients in the preceding equations. By definition

$$\bar{h} = \frac{(5 + \bar{M}^2)^3}{216\,\bar{M}} \qquad \text{(for } \gamma = 1.4\text{)}$$

and solving this equation from $\bar{M}$ in terms of $\bar{h}$, we obtain

$$\bar{M}^2 = 1 + 2\sqrt{\tfrac{6}{5}(\bar{h}-1)} + \tfrac{14}{9}\left[\tfrac{6}{5}(\bar{h}-1)\right] + \cdots$$

which for $\bar{h} = 1 + \xi^2$ becomes

$$M^2 = 1 + 2\sqrt{\tfrac{6}{5}}\,\xi + \tfrac{14}{9}\cdot\tfrac{6}{5}\cdot\xi^2 + \cdots$$

then, approximately

$$x_4 \cong -\left(0.121716 + 0.3852\,\xi + 12.7234\,\xi^2 + \cdots\right) \qquad (6')$$

$$y_5 \cong (1+\xi^2)\left(0.24000 + 0.96886\,\xi + 1.7737\,\xi^2 + \cdots\right) \qquad (9')$$

$$m_4 \cong 1.47778 + 3.0659\,\xi + \cdots \, . \qquad (11')$$

8. A sketch of the nozzle illustrating notation is given on Figure 1. The design of a nozzle is started at pt L, the point on the nozzle axis at which the final (test section) Mach number is first achieved. Let this Mach number be $\bar{M}_t = M_t$. The corresponding area ratio $\bar{h}_t$ can then be easily obtained. The coordinates of this point are seen to be $x = \xi = \xi_t$ where, for $\bar{h} = 1 + \xi^2$, $\xi_t = \sqrt{\bar{h}_t - 1}$, and $y = \eta = 0$. Using these values as initial conditions, we can compute the Mach line ML which is given by the equation

$$\frac{d\eta}{d\xi} = \frac{-\left[1 - \frac{\bar{M}^2}{5}\left(\frac{M^{*2}}{\bar{M}^{*2}} - 1\right)\right]^3}{\bar{h}\sqrt{(\bar{M}^2 - 1) + \frac{6}{5}\bar{M}^2\left(\frac{M^{*2}}{\bar{M}^{*2}} - 1\right)}} \, , \qquad (14)$$

9. This equation must be integrated numerically. The Kutta-Runge method, described in reference (c), can be used. Equation 14 is in the form

$$\frac{d\eta}{d\xi} = f(\xi, \eta) \,. \tag{14a}$$

One integrates step by step. First compute

$$k_1 = f(\xi_1, \eta_1)\Delta\xi \tag{15a}$$

$$k_2 = f\left(\xi_1 + \frac{\Delta\xi}{2}, \eta_1 + \frac{k_1}{2}\right)\Delta\xi \tag{15b}$$

$$k_3 = f\left(\xi_1 + \frac{\Delta\xi}{2}, \eta_1 + \frac{k_2}{2}\right)\Delta\xi \tag{15c}$$

$$k_4 = f(\xi_1 + \Delta\xi, \eta_1 + k_3)\Delta\xi \,. \tag{15d}$$

Then $$\Delta\eta = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \,, \tag{15e}$$

$\bar{M}$ as a function of $\sqrt{\bar{h}-1}$ can be obtained from Table I and $\frac{M^*}{\bar{M}^*}$ from equation 3.

10. The integration is extended to a suitable point as previously discussed. Holding the value of η at this point constant, we can obtain from equations 1 and 2 the contour of the nozzle, a streamline, upstream of this point. The contour should be extended approximately again as far as the minimum section. The coordinates of the minimum section can be found from the relation

$$\xi_{\left(\frac{dy}{d\xi}=0\right)} \cong \frac{-\frac{1}{3}\sqrt{\frac{6}{5}}\,\eta^2 + 0.096885\,\eta^4}{1 + \frac{2}{3}\sqrt{\frac{6}{5}}\,\eta^2 + 0.24177\,\eta^4} \,, \tag{16}$$

One should note that for $\xi = 0$, $x \cong -0.121716\,\eta^4$.

11. The coordinates x, y of the remaining (terminal) portion of the nozzle can be determined from the following equations (see Figure 1):

$$x = x_1 + l\cos(\theta_1 + \alpha_1) \tag{17}$$

$$y = y_1 + l\sin(\theta_1 + \alpha_1) \tag{18}$$

where $$l = (\eta_i - \eta_1)\left(\frac{M^2+5}{6}\right)^3 , \qquad (19)$$

$$\alpha = \sin^{-1}\frac{1}{M} , \qquad (20)$$

$$M^2 = \frac{\bar{M}^2\left(\frac{M^{*2}}{\bar{M}^{*2}}\right)}{1 - \frac{\bar{M}^2}{5}\left(\frac{M^{*2}}{\bar{M}^{*2}} - 1\right)} , \qquad (21)$$

and θ can be obtained from equation (4). The subscripts "1" refer to the variable pt P on the line ML and "i" refers to the "inflexion" point M.

J. C. Crown

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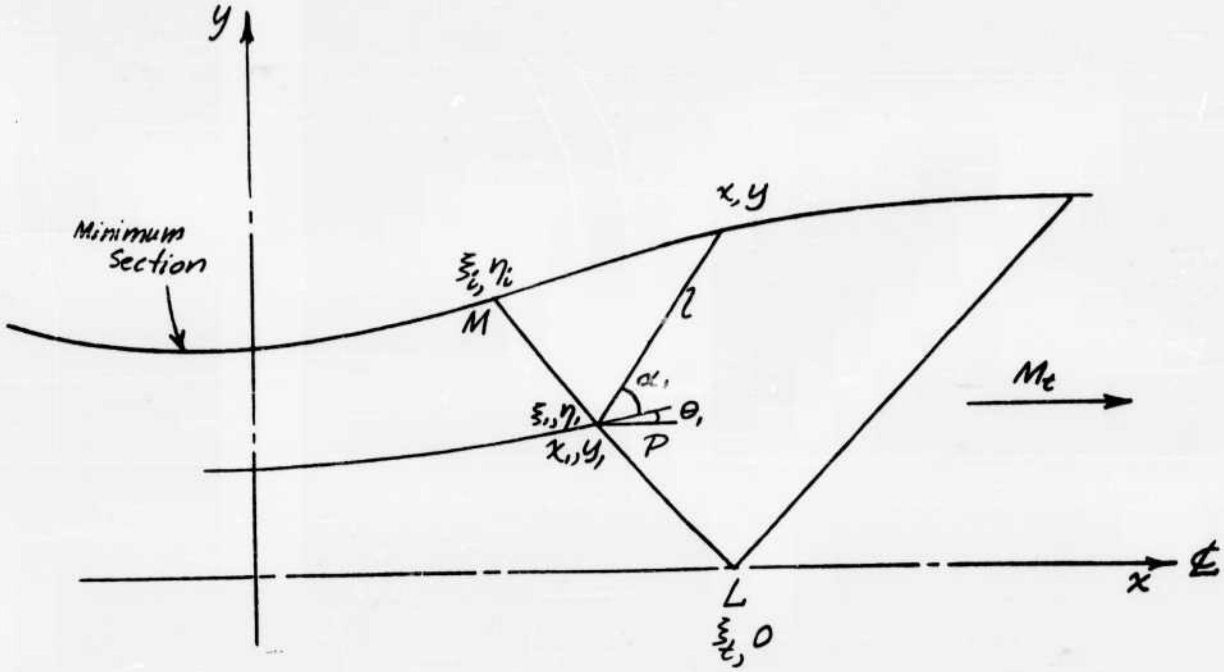

Fig 1. Variables used in Nilson method (Refs. a&b)

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Table I *

$\bar{M}$ as a Function of $\sqrt{\bar{h}-1}$

$\sqrt{\bar{h}-1}$	$\bar{M}$	$\sqrt{\bar{h}-1}$	$\bar{M}$	$\sqrt{\bar{h}-1}$	$\bar{M}$	$\sqrt{\bar{h}-1}$	$\bar{M}$
.00	1.00000	.43	1.51276	.87	2.04793	.00	1.00000
.01	1.01099	.44	1.52519	.88	2.05959	-.01	.98908
.02	1.02204	.45	1.53761	.89	2.07122	-.02	.97822
.03	1.03316	.46	1.55004	.90	2.08283	-.03	.96744
.04	1.04434	.47	1.56246	.91	2.09440	-.04	.95673
.05	1.05558	.48	1.57488	.92	2.10595	-.05	.94609
.06	1.06688	.49	1.58730	.93	2.11746	-.06	.93552
.07	1.07824	.50	1.59971	.94	2.12894	-.07	.92502
.08	1.08965	.51	1.61212	.95	2.14040	-.08	.91461
.09	1.10112	.52	1.62452	.96	2.15182	-.09	.90426
.10	1.11265	.53	1.63691	.97	2.16321	-.10	.89398
.11	1.12422	.54	1.64929	.98	2.17457	-.11	.88381
.12	1.13585	.55	1.66166	.99	2.18590	-.12	.87370
.13	1.14752	.56	1.67403	1.00	2.19720	-.13	.86367
.14	1.15925	.57	1.68638	1.01	2.20847	-.14	.85372
.15	1.17102	.58	1.69872	1.02	2.21970	-.15	.84386
.16	1.18283	.59	1.71104	1.03	2.23091	-.16	.83408
.17	1.19469	.60	1.72336	1.04	2.24208	-.17	.82438
.18	1.20658	.61	1.73565	1.05	2.25322	-.18	.81476
.19	1.21852	.62	1.74793	1.06	2.26433	-.19	.80523
.20	1.23050	.63	1.76020	1.07	2.27541	-.20	.79578
.21	1.24251	.64	1.77244	1.08	2.28645	-.21	.78642
.22	1.25455	.65	1.78467	1.09	2.29747	-.22	.77715
.23	1.26663	.66	1.79688	1.10	2.30845	-.23	.76796
.24	1.27874	.67	1.80907	1.11	2.31940	-.24	.75886
.25	1.29008	.68	1.82124	1.12	2.33032	-.25	.74985
.26	1.30305	.69	1.83339	1.13	2.34120	-.26	.74093
.27	1.31524	.70	1.84552	1.14	2.35206	-.27	.73209
.28	1.32746	.71	1.85763	1.15	2.36288	-.28	.72334
.29	1.33969	.72	1.86971	1.16	2.37367	-.29	.71469
.30	1.35197	.73	1.88177	1.17	2.38443	-.30	.70612
.31	1.36426	.74	1.89380	1.18	2.39516		
.32	1.37656	.75	1.90502	1.19	2.40585		
.33	1.38889	.76	1.91780	1.20	2.41651		
.34	1.40123	.77	1.92977	1.21	2.42715		
.35	1.41358	.78	1.94170	1.22	2.43725		
.36	1.42595	.79	1.95362	1.23	2.44831		
.37	1.43832	.80	1.96550	1.24	2.45885		
.38	1.45071	.81	1.97736	1.25	2.46936		
.39	1.46311	.82	1.98919	1.26	2.47983		
.40	1.47552	.83	2.00099	1.27	2.49027		
.41	1.48793	.84	2.01277	1.28	2.50068		
.42	1.50034	.85	2.02452	1.29	2.51106		
.43	1.51276	.86	2.03624	1.30	2.52141		
		.87	2.04793				

*This Table appeared in reference (a).